UNIT 10 (Extended) STATISTICS AND TRANSFORMATIONS

Recommended Prior Knowledge
It is strongly recommended that candidates have a thorough understanding of Unit 1, Unit 8 and Unit 9.

Context
This unit revises and develops mathematical concepts in Statistics and establishes a deeper understanding of Transformations. Candidates should use calculators where appropriate.

Outline
The topics in this unit may be studied sequentially. There is some element of choice, however, and Centres may wish to teach topics in a different order, for example Statistics need not be studied first. Basic ideas of vectors and matrices are studied together with various transformations of the plane. Candidates are also introduced to simple statistical concepts and diagrams. With all sections it is expected that candidates will be set questions of varying difficulty to complete for themselves. The unit gives candidates the opportunity to work investigatively and thus establish the skills needed for the submission of coursework.

	Learning Outcomes	Suggested Teaching Activities	Resources
33	Collect, classify and tabulate statistical data; read, interpret and draw simple inferences from tables and statistical diagrams; construct and use bar charts, pie charts, pictograms, simple frequency distributions, histograms with equal intervals and scatter diagrams (including drawing a line of best fit by eye); understand what is meant by positive, negative and zero correlation; calculate the mean, median and mode for individual and discrete data and distinguish between the purposes for which they are used; calculate the range Construct and read histograms with equal and unequal intervals (areas proportional to frequencies and vertical axis labelled ‘frequency density’); construct and use cumulative frequency diagrams; estimate and interpret the median, percentiles, quartiles and inter-quartile range; calculate an estimate of the mean for grouped and continuous data; identify the modal class from a grouped frequency distribution.	Use simple examples to revise collecting data and presenting it in a frequency (tally) chart. For example, record the different makes of car in a car park, record the number of letters in each of the first 100 words in a book, etc. Use the data collected to construct a pictogram, a bar chart and a pie chart. Point out that the bars in a bar chart can be drawn apart. **Class activity:** Design and use a questionnaire, collate results and present them in diagramatic form. From data collected show how to work out the mean, the median and the mode. Use simple examples to highlight how these averages may be used. For example in a discussion about average wages the owner of a company with a few highly paid managers and a large work force may wish to quote the mean wage rather than the median. Point out how the mode can be recognised from a frequency diagram. Use a simple example to show how discrete data can be grouped into equal classes. Draw a histogram to illustrate the data (i.e. with a continuous scale along the horizontal axis). Point out that this information could also be displayed in a bar chart (i.e. with bars separated). **Class activity:** Investigate the length of words used in two different newspapers and present the findings using	Try the ‘Bat Wings’ problem at http://nrich.maths.org/public/leg.php Compare the median and the mean interactively at http://www.standards.nctm.org/document/eexamples/chap6/6.6/index.htm Download newspaper stories - worldwide coverage at http://www.newsparadise.com/

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<tr><td></td><td></td><td>statistical diagrams.
Explain cumulative frequency and use a straightforward example to illustrate how a cumulative frequency table is constructed. Draw the corresponding cumulative frequency curve. Point out that this can be approximated by a cumulative frequency polygon.

Use a cumulative frequency curve to help explain percentiles. Introduce the names given to the 25th, 50th and 75th percentiles and show how to estimate these from a graph. Show how to calculate the range of a set of data and how to estimate the inter-quartile range from a cumulative frequency diagram.

Record sets of continuous data, e.g. heights, weights etc., in grouped frequency tables. Use examples that illustrate equal and unequal class widths. Draw the corresponding histograms (label the vertical axis of a histogram as ‘frequency density’ and point out that the area of each bar is proportional to the frequency). Show how to calculate frequencies from a given histogram and how to identify the modal class.

Use straightforward examples to show how to calculate an estimate for the mean of data in a grouped frequency table.

Class activity: Survey a class of students - heights, weights, number in family, etc. Use different methods of display to help analyse the data and make statistical inferences.</td><td></td></tr>
<tr><td>35</td><td>Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or a; add and subtract vectors; multiply a vector by a scalar.
Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$.

(Vectors will be printed as $\overrightarrow{AB}$ or a and their magnitudes denoted by modulus signs, e.g. $|\overrightarrow{AB}|$ or $|\mathbf{a}|$. In their answers to questions candidates are expected to indicate a in some definite way, e.g. by an arrow or by underlining, thus $\overrightarrow{AB}$ or $\underline{a}$).

Represent vectors by directed line segments; use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors; use position vectors.</td><td>Use the concept of translation to explain a vector. Use simple diagrams to illustrate column vectors in two dimensions, explaining the significance of positive and negative numbers. Introduce the various forms of vector notation.

Show how to add and subtract vectors algebraically and by making use of a vector triangle. Show how to multiply a column vector by a scalar and illustrate this with a diagram.

Use simple diagrams to help show how to calculate the magnitude of a vector (Pythagoras’ theorem may have to be revised).

Define a position vector and solve various straightforward problems in vector geometry.</td><td>Interactive work on vector sums at
http://www.standards.nctm.org/document/eexamples/chap7/7.1/part2.htm</td></tr>
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36	Display information in the form of a matrix of any order; calculate the sum and product (where appropriate) of two matrices; calculate the product of a matrix and a scalar quantity; use the algebra of 2x2 matrices including the zero and identity 2x2 matrices; calculate the determinant and inverse $\mathbf{A}^{-1}$ of a non-singular matrix **A**.	Use simple examples to illustrate that information can be stored in a matrix. For example, the number of different types of chocolate bar sold by a shop each day for a week. Define the order/size of a matrix as the number of rows x number of columns. **Class activity:** Investigate networks - recording information in a matrix. (This is not on the syllabus but it will broaden candidates mathematical knowledge of matrices) Explain how to identify matrices that you may add/subtract or multiply together. Use straightforward examples to illustrate how to add/subtract and multiply matrices together. Define the identity matrix and the zero matrix. Use simple examples to illustrate multiplying a matrix by a scalar quantity. Use straightforward examples to illustrate how to calculate the determinant and the inverse of a non-singular 2x2 matrix. **Class activity:** Investigate how to use matrices to help solve simultaneous equations.	
37	Reflect simple plane figures in horizontal or vertical lines; rotate simple plane figures about the origin, vertices or mid points of edges of the figures, through multiples of 90^o ; construct given translations and enlargements of simple plane figures; recognise and describe reflections, rotations, translations and enlargements. Use the following transformations of the plane: reflection (M); rotation (R); translation (T); enlargement (E); shear (H); stretching (S) and their combinations. (If M(a) = b and R(b) = c the notation RM(a) = c will be used; invariants under these transformations may be assumed). Identify and give precise descriptions of transformations connecting given figures; describe transformations using co-ordinates and matrices (singular matrices are excluded).	Draw an arrow shape (☑) on squared paper. Use this to illustrate: reflection in a line (mirror line), rotation about any point (centre of rotation) through multiples of 90^o (in both clockwise and anti-clockwise directions) and translation by a vector. Several different examples of each translation should be drawn. Use the word image appropriately. **Class activity:** Using a pre-drawn shape on (x,y) coordinate axes to complete a number of transformations using the equations of lines to represent mirror lines and coordinates to represent centres of rotation. Work with (x,y) coordinate axes to show how to find: the equation of a mirror line given a shape and its (reflected) image, the centre and angle of rotation given a shape and its (rotated) image, the vector of a translation. Draw a triangle on squared paper. Use this to illustrate enlargement by a positive integer scale factor about any point (centre of enlargement). Show how to find the centre of enlargement given a shape and its (enlarged) image. Draw straightforward enlargements using negative and/or fractional (½) scale factors. Show how to calculate the area of an image after enlargement by scale factor k. Use straightforward examples to illustrate a shear and a stretch. Using a shape and its image drawn on (x,y)	For further information about transformations search for 'rotation', 'enlargement', 'reflection' or 'translation' at http://www.learn.co.uk

		coordinate axes show how to find the scale factor and the equation of the invariant line. **Class activity:** Starting with a letter E drawn on (x,y) coordinate axes, perform combinations of the following transformations: translation, rotation, reflection, stretch, shear and enlargement. Use a unit square and the base vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ to identify matrices which represent the various transformations met so far, e.g. $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ represents a rotation about (0,0) through 90^o anti-clockwise. Work with a simple object drawn on (x,y) coordinate axes to illustrate how it is transformed by a variety of given matrices. Use one of these transformations to illustrate the effect of an inverse matrix. Work with a rectangle drawn on (x,y) coordinate axes to illustrate that the area scale factor of a transformation is numerically equal to the determinant of the transformation matrix. For example use the matrix $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$.	